Capillary rise of a meniscus with phase change

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ABSTRACT

The Lucas–Washburn equation, describing the motion of a liquid body in a capillary tube, is extended to account for the effect of phase change – evaporation or condensation. The system is found to always possess a stable equilibrium state when the temperature jump across the interface is confined to a certain range. We show that phase change affects the equilibrium height of the meniscus, the transition threshold from monotonic to oscillatory dynamics, and the frequency of oscillations, when present. At higher mass transfer rates and/or large capillary radii, vapor recoil is found to be the dominant factor. Evaporation lowers the equilibrium height, increases the oscillation frequency and lowers the transition threshold to oscillations. For condensation, two regimes are identified: at high mass transfer rates similar trends to those of evaporation are observed, whereas the opposite is found for low mass transfer rates, resulting in an increased equilibrium height, lower oscillation frequencies and a shift of the transition threshold toward monotonic dynamics.

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1. Introduction

Capillary-driven flow is a long standing problem encompassing hydrodynamics, contact line dynamics (wetting) and surface phenomena. Its occurrence is widespread and it is found in the natural environment, such as soil–water transport, plant transpiration, as well as in engineered systems, e.g. fuel cells, heat pipes of loop and other types. In particular, when a capillary of a sufficiently small radius is brought in contact with a "wetting" fluid, the fluid spontaneously enters the capillary and flows, displacing the fluid already occupying the capillary. According to a recent review [1], the problem of capillary rise in a cylindrical tube dates back to the mid-18th century, addressed to Euler as an example of Newtonian dynamics with a variable mass. Euler's response is unknown, and it was not until the pioneering works of Lucas [2], Washburn [3] and Bosanquet [4], that a mathematical model was formulated and analysis of various limiting cases of the problem was presented.

Within the framework of the Lucas–Washburn equation, a force balance on a liquid body is considered, accounting for the forces due to capillarity, gravity, viscous drag and inertia. Since then, numerous theoretical and experimental studies have been dedicated to probing various characteristics of capillary rise, see for instance [1,5] and references therein. A primary simplification made in this approach is the assumption of a fully developed laminar flow, allowing the use of the well-known Poiseuille law to account for the

viscous drag. Various improvements and extensions to the theory have thus been primarily concerned with end effects, e.g. developing flow and energy dissipation at the capillary entrance, as well as contact line dynamics, e.g. contact angle hysteresis, friction and meniscus evolution [5–9]. Another extension considered an additional viscous drag due to the displaced gas in the capillary [7].

In the last decade, the inertial regime of the capillary flow has received some attention, and the existence of oscillations was demonstrated, both theoretically [10] and experimentally [7,11]. The characteristics of this inertial, oscillatory regime have been analyzed for inviscid and viscous fluids [6], illustrating the dependence of the frequency and amplitude on the physical parameters of the liquid, as well as the entrance effects which, due to the flow reversal, have an even greater impact on the flow than usual.

The effect of interfacial mass transfer, due to phase change by either evaporation or condensation, on the capillary rise has thus far, to the best of our knowledge, not been investigated within the Lucas–Washburn framework. The possibility of an increased liquid viscosity due to evaporative cooling was considered by Zhmud et al. [7] as a possible source of discrepancy with experimental observations. However, this was ruled out as an isothermal, vapor-saturated, experimental system should not permit sufficient evaporation to justify the estimated increase of viscosity. The evolution of an evaporating pinned meniscus due to a step change in the evaporation rate was studied by Rand [12], who concluded that the meniscus may experience damped oscillations under appropriate conditions. The transition values for the emergence of oscillations were found in terms of a single dimensionless parameter, which strongly depends upon the capillary radius and liquid

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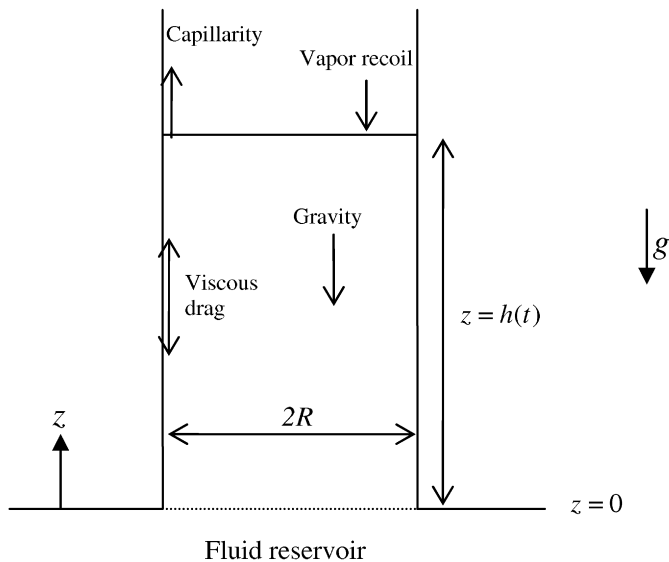*E-mail address:* meroron@technion.ac.il (A. Oron).

Fig. 1. Sketch of the system.

viscosity. For instance, in the case of water in a 1-m-long capillary, this transition to meniscus oscillations occurs for a radius larger than approximately 166 µm. Although the analysis [12] was not made within the Lucas–Washburn model, it is nevertheless a related problem and illustrates the possible effects imparted by interfacial mass transfer.

Normally, phase change at the interface creates large accelerations of the vapor due to a large disparity in liquid and vapor densities, and the backward reaction is known as vapor recoil. The pressure exerted by vapor recoil on the interface is directed into the liquid phase for both evaporation and condensation [13–15]. Hickman [16] found that vapor recoil can be very important in terms of the behavior of the gas–liquid interface where phase change takes place, especially under conditions of reduced pressure.

It is the purpose of this paper to extend the Lucas–Washburn theory so as to incorporate the effect of interfacial phase change on the capillary flow dynamics. The paper is organized as follows: Section 2 presents the model formulation, in Section 3 we present some numerical results and discuss the various characteristics of the flow, whereas Section 4 presents a discussion of the results. A summary of the work is given in Section 5.

2. Model formulation

We consider a long vertical capillary of radius R, brought in contact with a large reservoir of a liquid with density ρ and viscosity μ shown schematically in Fig. 1. The liquid is assumed to completely wet the capillary inner wall, hence the contact angle θ is set equal to zero. The liquid is kept at a constant temperature, which may be above or below a reference saturation temperature of the vapor within the capillary by the value ΔT. Due to the temperature difference, evaporation or condensation will occur at the moving vapor–liquid interface.

We extend the equation of motion describing the rise of the liquid body within the capillary, proposed by Lucas [2] and Washburn [3],

$$\rho \frac{d}{dt}(hu) = \frac{2\sigma}{R} - \rho g h - \frac{8\mu}{R^2} hu + p_v \qquad (1)$$

to account for the possibility of phase change at the vapor–liquid interface. This equation represents the balance of forces per unit area of the horizontal cross-section, acting on the liquid body in

the vertical direction. Here, t is time, $h(t)$ is a cross-sectional averaged interfacial height with respect to the level of the liquid in the reservoir, u is the average axial velocity of the fluid, σ is the vapor–liquid surface tension, g is the gravitational acceleration and p_v is the dynamic pressure exerted by the vapor on the interface, often referred to as vapor thrust or recoil. This pressure is the result of the velocity jump which the vapor undergoes during evaporation/condensation. Since the phase change creates a density jump, a corresponding velocity jump is necessary for the mass flux to be continuous at the interface

$$\rho \left(u - \frac{dh}{dt}\right) = \rho_v \left(u_v - \frac{dh}{dt}\right) = j, \qquad (2)$$

where j is the interfacial mass flux due to evaporation/condensation, and the subscript v denotes the corresponding properties of the vapor.

The kinematic boundary condition at the interface requires [15] that

$$u - \frac{dh}{dt} = \frac{j}{\rho}. \qquad (3)$$

Equation (3) states that the position of the interface depends on the rate of interfacial mass transfer relative to the capillary-driven flow.

The momentum flux jump condition at the interface, in conjunction with Eq. (2), can be expressed as [15]

$$\rho u \left(u - \frac{dh}{dt}\right) - \rho_v u_v \left(u_v - \frac{dh}{dt}\right) = j(u - u_v) = -\frac{\rho - \rho_v}{\rho \rho_v} j^2, \qquad (4)$$

where the last term on the right-hand side represents vapor recoil.

Since $\rho_v \ll \rho$, it is possible to simplify the vapor recoil term in Eq. (4) as

$$p_v = -\frac{j^2}{\rho_v}. \qquad (5)$$

Equation (1) can now be rewritten as

$$\rho \frac{d}{dt}\left(h \frac{dh}{dt} + h \frac{j}{\rho}\right) = \frac{2\sigma}{R} - \rho g h - \frac{8\mu}{R^2}\left(h \frac{dh}{dt} + h \frac{j}{\rho}\right) - \frac{j^2}{\rho_v}. \qquad (6)$$

It is worthwhile noting that in the governing equation (6), the vapor recoil term always induces a pressure in the negative direction, opposing the capillary flow, regardless of whether evaporation/condensation is present.

Several features of the flow considered here can be emphasized. First, when phase change does not take place, $j = 0$, the interfacial motion is driven by capillarity and is governed by the Lucas–Washburn equation (1) without the recoil term, where $u = dh/dt$. Next, in the presence of evaporation (condensation), $j > 0$ ($j < 0$), and therefore the motion of the interface will be slower (faster) than with no phase change. Finally, despite the presence of phase change, a steady state of the interface may be achieved being exactly balanced by all factors involved.

At this point it must be noted that both evaporation and condensation influence the temperature field in the liquid, causing a local decrease and increase, respectively, of the interfacial temperature due to the latent heat of vaporization $\mathcal{L}$. However, a full analysis of the heat transfer and fluid flow near the interface [17, 18] requires the solution of the flow field in the vicinity of the meniscus and contact line, and is therefore beyond the scope of this paper. Such analysis would invalidate the use of the Lucas–Washburn theory, in which the meniscus shape is considered to be flat. Within this simplified framework, it may be possible to consider the heat transfer by introducing an empirical heat transfer coefficient, in a way similar to the contact-line friction factor introduced by Hamraoui and Nylander [8]. We therefore assume,

Table 1
Physical properties of water and diethyl ether, at 25 °C, compiled from Ref. [20]

	ρ (kg/m^3)	μ (kg/m s)	σ (N/m)	$\mathcal{L}$ (kJ/kg)	M_w (kg/mol)
Water	997.0	9.11×10^{-4}	0.072	2360.0	0.018
Ether	708.0	2.23×10^{-4}	0.016	377.0	0.074

for the purpose of this study, that the temperature in the liquid is kept constant by some external means, say by heating the capillary wall. The phase change occurring at a constant rate is then driven by a temperature jump at the interface ΔT. The resulting mass flux may be approximated using kinetic theory which in linearized form reads [13–15]

$$j = k(T^I - T_v) \tag{7}$$

where T^I, T_v denote the temperature at the interface and in the vapor, respectively, and

$$k = \frac{\alpha \rho_v \mathcal{L}}{T_v^{3/2}} \left(\frac{M_w}{2\pi R_g} \right)^{1/2}. \tag{8}$$

Here R_g is the universal gas constant, M_w is the molecular mass of the liquid, and α is the accommodation coefficient, taken here as unity. The forthcoming study uses water and diethyl ether as model liquids and their material properties are given in Table 1. Diethyl ether is chosen for comparison with water due to its lower surface tension, density and viscosity but higher volatility.

3. Equilibrium state and its stability

Equation (6) differs from the Lucas–Washburn equation through the addition of the interfacial mass flux which enters the inertial and viscous terms, as well as by the presence of the vapor recoil term.

We first look for equilibria of the system. Under steady state conditions, the equation of motion (6) reduces to

$$\frac{2\sigma}{R} - \rho g H - \frac{8\mu}{R^2} \frac{j}{\rho} H - \frac{j^2}{\rho_v} = 0, \tag{9}$$

where H is the equilibrium position of the interface, which can be easily obtained as

$$H = \frac{F_{ca} - F_r}{F_v u_0 + g}. \tag{10}$$

Here we have made the following definitions:

$$F_{ca} = \frac{2\sigma}{\rho R}, \qquad F_r = \frac{\rho}{\rho_v} u_0^2, \qquad F_v = \frac{8\mu}{\rho R^2} \tag{11}$$

representing the contributions of capillarity, vapor recoil, and viscous drag, respectively. We also make use of the fact that the steady state velocity is given by $u_0 = j/\rho$, as follows from the kinematic boundary condition (3).

In the case where no interfacial mass transfer is present, Eq. (9) reduces to the familiar balance of gravity and capillarity [19]. The presence of interfacial mass transfer alters the equilibrium position, as illustrated in Fig. 2, which shows the equilibrium height of the meniscus H normalized by the equilibrium height in the isothermal case H_0. Evaporation always lowers the equilibrium height, as expected, while condensation may result in an either lower or higher position, depending on the mass flux expressed here in terms of the temperature difference across the meniscus interface ΔT. The interfacial mass flux manifests through two separate mechanisms, namely vapor recoil and mass loss/gain. Vapor recoil always tends to lower the equilibrium height. However, in the case of a sufficiently low condensation rate, the mass gain may
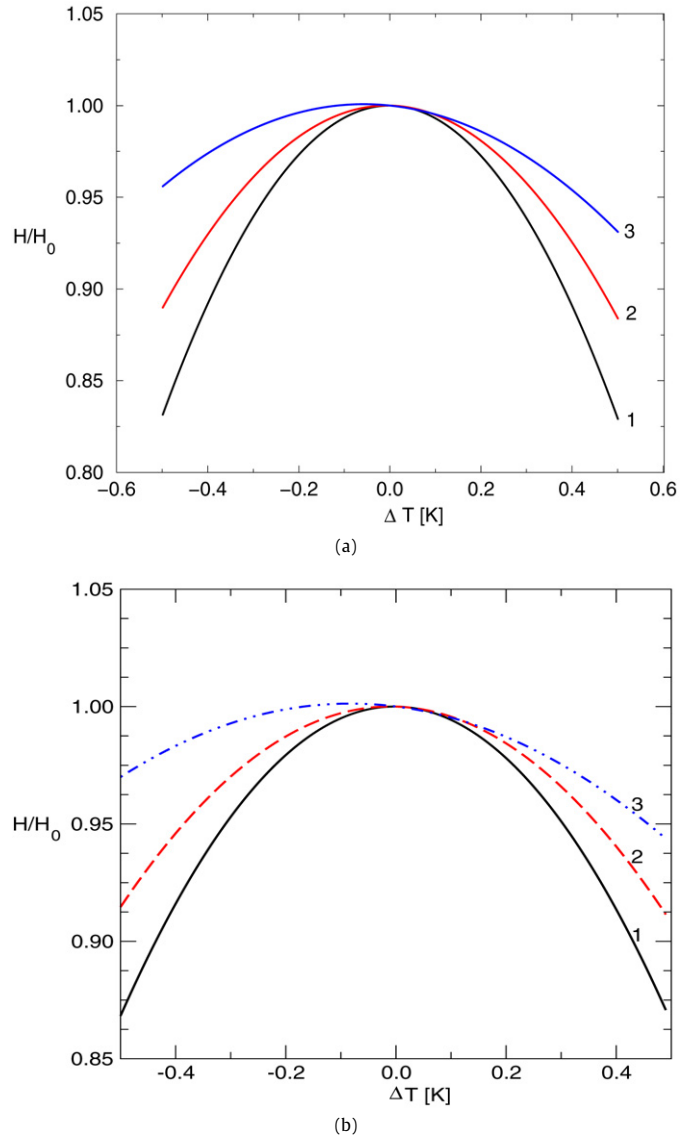

Fig. 2. Equilibrium position of the meniscus H scaled with its equilibrium height in the isothermal case H_0, as a function of the temperature difference across the interface ΔT. (a) Water: 1, $R = 1.5$ mm; 2, $R = 1$ mm; 3, $R = 0.5$ mm. (b) Diethyl ether: 1, $R = 0.9$ mm; 2, $R = 0.6$ mm; 3, $R = 0.3$ mm.

raise the equilibrium position to the point where vapor recoil becomes the dominant mechanism and the equilibrium position is lowered. This effect is much more noticeable for diethyl ether due to the fact that its vapor density is larger than that of water, resulting in a corresponding decrease of vapor recoil, as follows from Eq. (5).

We now consider the stability of the steady interfacial position H by introducing a small disturbance to the equilibrium in the form

$$h = H + \varepsilon(t), \tag{12}$$

where $\varepsilon(t)$ represents a small departure of the interface elevation from its equilibrium.

Substituting Eq. (12) into Eq. (6) and linearizing the latter with respect to $\varepsilon(t)$, we obtain the following equation:

$$\varepsilon''(t) + \alpha \varepsilon'(t) + \beta \varepsilon(t) = 0 \tag{13}$$

where primes denote differentiation with respect to time, and

$$\alpha = \frac{u_0}{H} + F_v, \qquad \beta = \frac{g}{H} + \frac{u_0}{H} F_v. \tag{14}$$
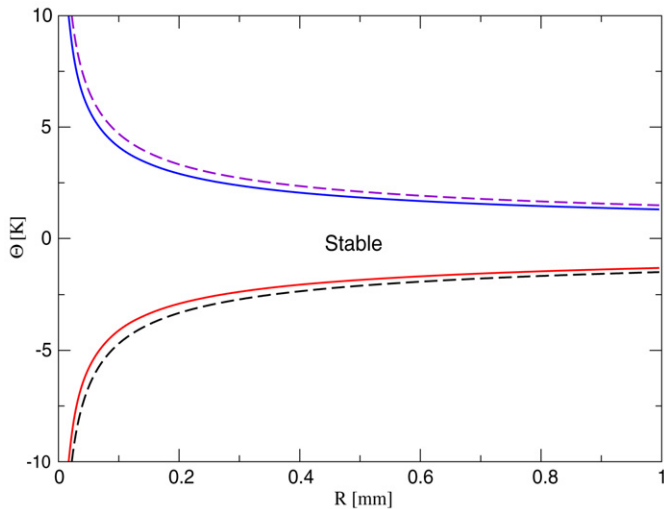
Fig. 3. The maximal temperature jump across the interface which allows the stability of the equilibrium height H, as a function of the capillary radius R. The solid and dashed curves correspond to diethyl ether and water, respectively. The values of $\theta = \max \Delta T$ shown here represent the limitation of the current model.

The (asymptotic) stability criterion for Eq. (13) requires that $\alpha > 0$ and $\beta > 0$ which is equivalent to

$$j > -\rho \max\left(HF_v, \frac{g}{F_v}\right). \qquad (15)$$

It is immediately deduced from Eq. (10) that when no interfacial mass transfer is present, i.e. $u_0 = 0$, the equilibrium is unconditionally stable, since in that case $\alpha \equiv F_v > 0$, $\beta \equiv g/H > 0$.

It also follows from Eqs. (10) and (14) that for $\Delta T \neq 0$, the equilibrium height H vanishes at the same time when α is singular, and these events take place when $F_r = F_{ca}$. This is in fact a singular point of Eq. (6), as it implies that the interfacial height passes through zero and may become negative. Also, when $F_r > F_{ca}$, the equilibrium height becomes negative. Thus, for $F_r \geqslant F_{ca}$ the Lucas–Washburn model breaks down.

The equilibrium height of the system is affected by the phase change, which either lowers or raises it. While evaporation always lowers the equilibrium position, low rates of condensation will raise it, up to the point where vapor recoil becomes the dominant mechanism and lowers the equilibrium height. At sufficiently high mass transfer rates, the equilibrium position can formally become negative which is physically unacceptable. The states with $h \leqslant 0$ represent the situations where a momentum conservation law, leading to the Lucas–Washburn equation (1), is formulated for a body of a zero or negative mass, which does not have any physical meaning. This happens when $\alpha \leqslant 0$. We find that the equilibrium height H of the meniscus given by Eq. (10) is stable as long as $H > 0$. Fig. 3 displays the stability limit determined by $\alpha = 0$ in terms of the maximal temperature jump across the interface, as a function of the capillary radius R. When ΔT is outside the range delineated by the appropriate curves in Fig. 3, the solutions of Eq. (6) pass through $h = 0$ and the model loses its validity. It should be noted, however, that this limitation exists not only for the present theory but also for the Lucas–Washburn approach in general even in the isothermal case. As an example, if a liquid body is released from an arbitrary height much above its equilibrium position, the liquid inertia will cause the interface to pass through zero elevation, and the theory is invalidated again.

When a stable equilibrium exists, Eq. (13) predicts the following two possibilities for the system's approach to it in the "long-time" limit: when the discriminant $\delta \equiv \alpha^2 - 4\beta > 0$, the system is over-

damped and a monotonic behavior is expected, whereas for $\delta < 0$, damped oscillations occur.

In the case where no interfacial mass transfer is present, the transition from the oscillatory to the monotonic evolution of the interface occurs at $F_v = 2\sqrt{g/H}$. At this point, the viscous drag is sufficient to suppress any "overshoot" due to the liquid inertia. It is worthwhile noting, that $\sqrt{g/H}$ corresponds to the natural frequency of an inviscid-fluid meniscus with no phase change, as follows from Eqs. (13) and (14).

The transition threshold determined by $\delta = 0$, namely

$$\delta_0 = F_v^2 - \frac{4g}{H}, \qquad (16)$$

corresponds for a given liquid to a critical value of the capillary radius, e.g., $R \approx 0.47$ mm for water. Finally, we note that the frequency of the system oscillations is given by $\Omega = \sqrt{\beta - \alpha^2/4}$.

In the following section, the results of the numerical study are presented to illustrate the various aspects of the capillary rise, as they emerge from the described model.

4. Results and discussion

Equation (6) is solved numerically using the fourth-order Runge–Kutta method for the physical parameters corresponding to the two model liquids, namely water and diethyl ether, specified in Table 1. Equation (6) is a second-order differential equation, thus it requires two initial conditions. It is clear that Eq. (6) has a "mathematical" singularity at $h = 0$. However, this singularity is not only a mathematical one but also an essential, physical singularity already discussed above. Due to this singularity a small value $0 < \epsilon \ll 1$ is assigned to the meniscus height for the numerical solution to begin: $h(0) = \epsilon$ and $h'(0) = 0$.

The time evolution of the interfacial elevation is shown in Figs. 4 and 5, calculated for water and ether, respectively, and for various values of the capillary radius R. The effect of the temperature difference which induces the phase change, is apparent and alters both the amplitude and frequency of the interfacial oscillations, when these are present. For any given capillary radius, ether oscillates faster than water, possibly due to the lower surface tension which results in a smaller amplitude of motion. It may be also seen in Figs. 4 and 5a that the water oscillations decay faster due to the stronger damping effect of the higher viscosity of water.

Figs. 4 and 5 also show the differences between the interfacial evolution of the evaporating and condensing cases. Figs. 4a and 5b show that the difference between the evaporating and condensing cases is small. This implies that the additional force due to vapor recoil is the dominant mechanism there, rather than the mass gain/loss.

As was previously stated, the vapor recoil induces a downward force for both evaporation and condensation, so that without it any difference in the evolution of the interface between the two cases can be attributed to mass gain/loss. The effect of the mass loss/gain is better seen in Figs. 6 and 7. The former shows the net effect of mass gain and loss on the interfacial evolution, whereas the latter presents the relative deviation $\eta = h(t)/h(t; \Delta T = 0)$ of the meniscus height calculated without the vapor-recoil term from that of the corresponding case with no interfacial mass transfer. When only mass loss/gain is considered, the relative deviation of the interface in the evaporation/condensation cases is exactly out of phase and is slightly skewed toward condensation. This skewness is a result of the different effect that mass loss/gain impart on the flow. While the decrease of the meniscus height due to the evaporated liquid is compensated by capillarity, the increase of the meniscus level due to condensation is opposed by gravity. However, the contribution of the added mass to the total liquid mass is
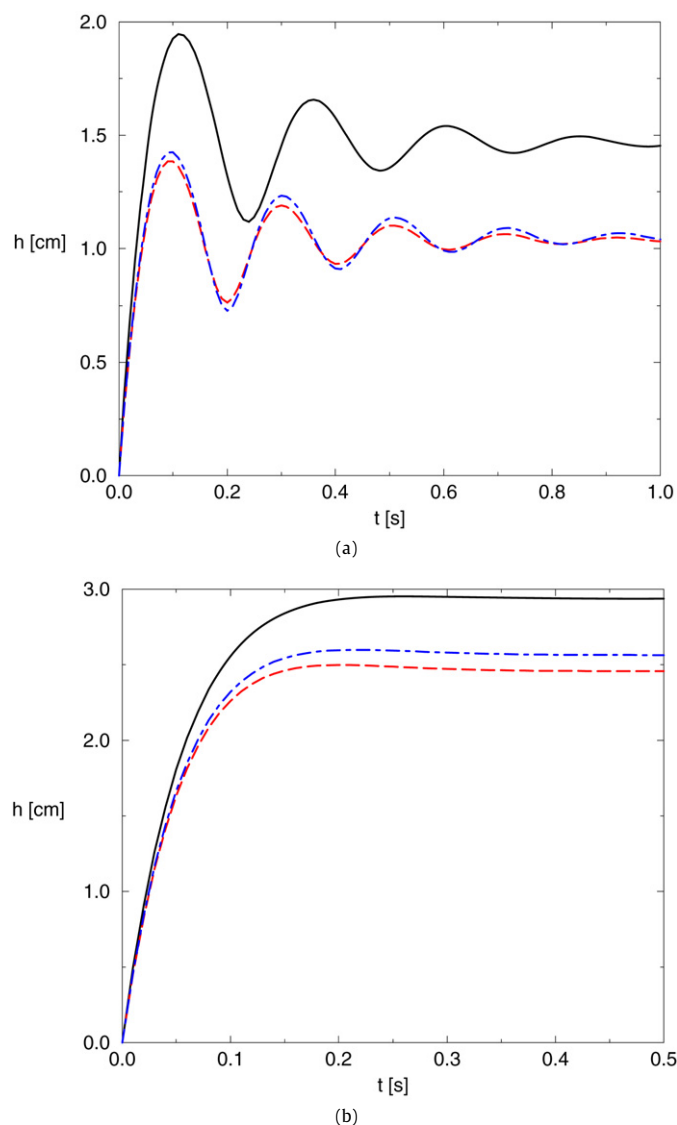
Fig. 4. Time evolution of the interfacial elevation h in the case of water. (a) $R = 1$ mm: the solid curve, $\Delta T = 0$ (no phase change); the dashed curve, the evaporating case with $\Delta T = 0.8$ K; the dot-dashed curve, the condensing case with $\Delta T = -0.8$ K. (b) $R = 0.5$ mm: the solid curve, $\Delta T = 0$ (no phase change); the dashed curve, the evaporating case with $\Delta T = 0.8$ K; the dot-dashed curve, the condensing case with $\Delta T = -0.8$ K.
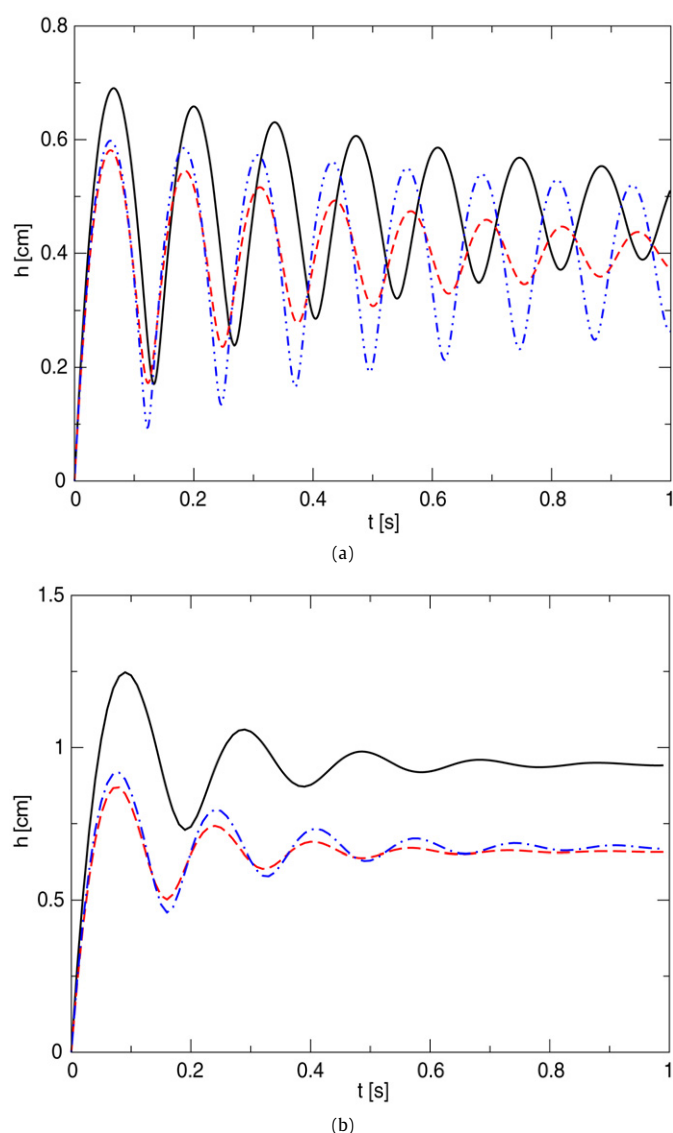
Fig. 5. Time evolution of the interfacial elevation h for diethyl ether. (a) $R = 1$ mm: the solid curve, $\Delta T = 0$ (no phase change); the dashed curve, the evaporating case with $\Delta T = 0.5$ K; the dot-dashed curve, the condensing case with $\Delta T = -0.5$ K. (b) $R = 0.5$ mm: the solid curve, $\Delta T = 0$ (no phase change); the dashed curve, the evaporating case with $\Delta T = 1$ K; the dot-dashed curve, the condensing case with $\Delta T = -1$ K.

small. Thus, the net condensation effect is a larger meniscus elevation during its rise and a correspondingly lower meniscus height during its descent. In fact, the skewness is stronger for ether than for water, since ether is lighter and therefore less affected by gravity. This mechanism also serves to explain why, in general, the differences between evaporation and condensation are larger for ether than for water, as seen in Figs. 4 and 5a. The mass lost through evaporation is not as strongly replenished by capillarity (lower surface tension), whereas mass gained by condensation has a smaller contribution to the total weight of the liquid body (lower density).

The parameter δ, which determines whether the dynamics of the meniscus interface is monotonic or oscillatory, is determined for water and diethyl ether as a function of the interfacial temperature jump, ΔT, for the capillary radius corresponding to the isothermal critical radius of transition from monotonic to oscillatory flow, see Fig. 8a. Evaporation always leads to oscillatory dynamics, due to a negative shift in the value of δ, while condensation induces a monotonic behavior for low values of ΔT, which

turns to oscillatory for higher temperature differences. This characteristic behavior sheds some light on the diverse effects on the dynamics of the interface of the two mechanisms emerging in the presence of interfacial mass transfer, namely vapor recoil and mass loss/gain. Vapor recoil induces a retarding force which always acts to lower the meniscus height, thus allowing a possible "overshoot" at smaller capillary radii, despite a stronger viscous damping. Since condensation adds mass to the liquid body, a slightly positive shift in the equilibrium height eliminates, at low ΔT, a possible "overshoot" which leads to oscillations. At higher mass flux rates, this effect becomes negligible as compared to the recoil force and oscillations are predicted, see Figs. 8b and 8c. As an illustrative example, Fig. 8d shows the effect of a high evaporation rate on the dynamics of a water body in a capillary of $R = 0.47$ mm, roughly corresponding to the radius at which a transition from monotonic to oscillatory dynamics is predicted in the isothermal case. As can be seen, under the effect of evaporation, a distinct oscillation is observed, while the corresponding isothermal dynamics is monotonic.
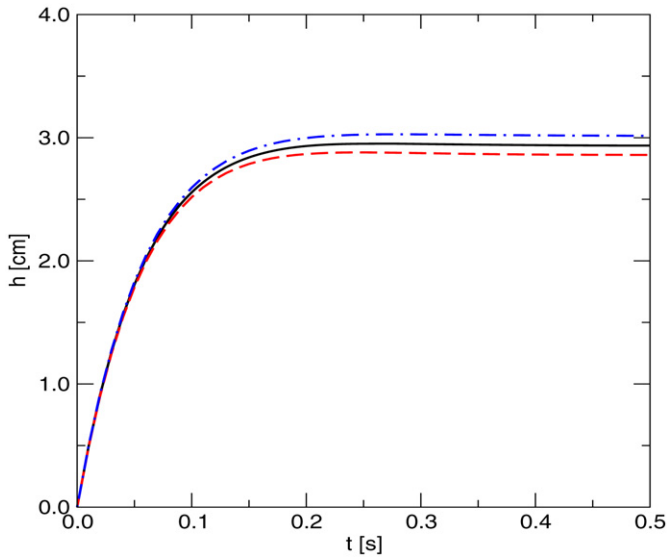
Fig. 6. The interfacial elevation $h(t)$ as a function of time t calculated from Eq. (6) without the vapor recoil term for water with $R = 0.5$ mm. The solid, dashed and dot-dashed curves correspond to $\Delta T = 0$—no phase change, the evaporating case $\Delta T = 1$ K, and the condensing case $\Delta T = -1$ K, respectively.
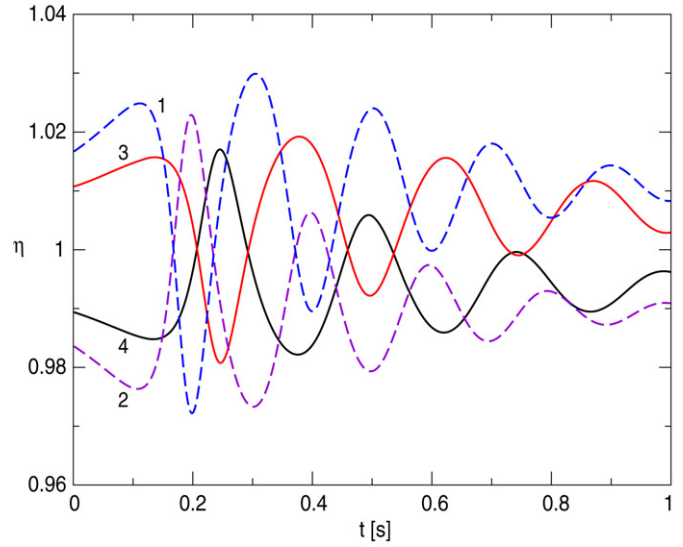

Fig. 7. Time evolution of the relative deviation of the interfacial elevation normalized with respect to that for $\Delta T = 0$, $\eta = h(t)/h(t; \Delta T = 0)$ determined from Eq. (6) without vapor recoil. The dashed curves correspond to ether with $R = 0.5$ mm: 1, $\Delta T = -1$ K; 2, $\Delta T = 1$ K. The solid curves correspond to water with $R = 1$ mm: 3, $\Delta T = -1$ K; 4, $\Delta T = 1$ K.
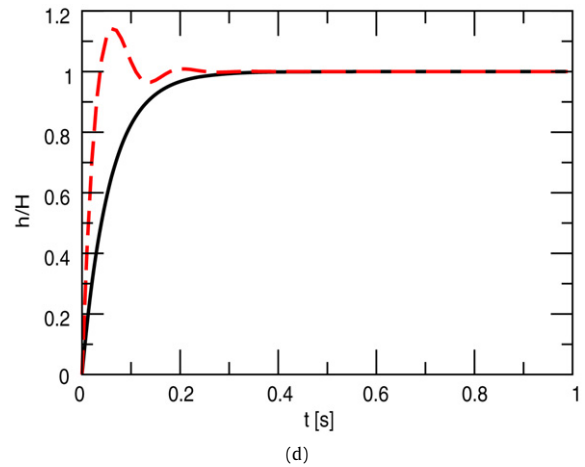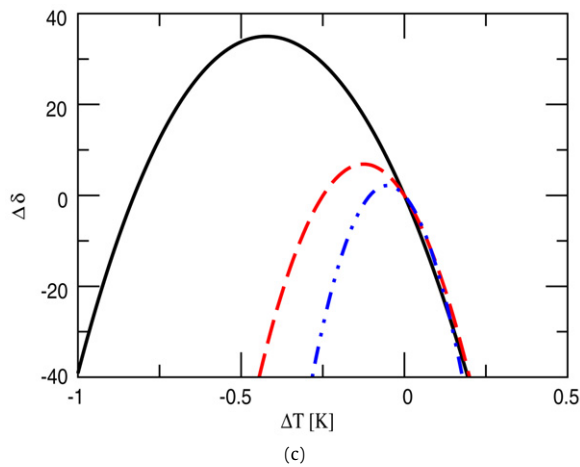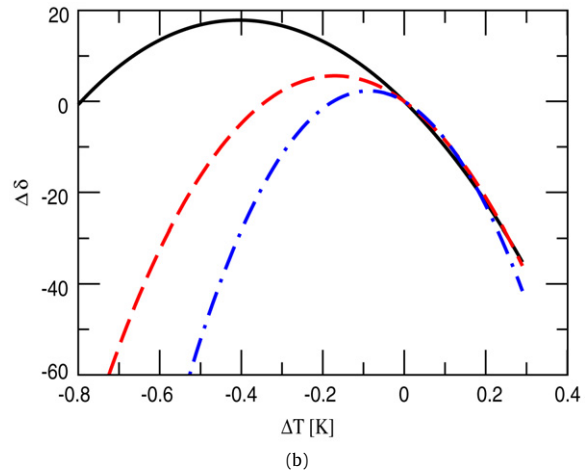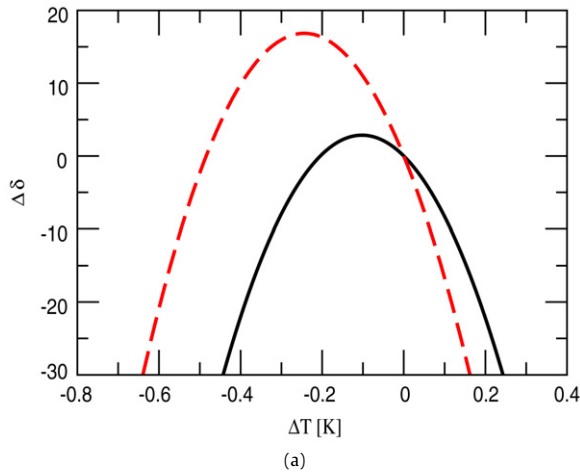

(a)

(b)

(c)

(d)

Fig. 8. (a) The contribution of phase change to the discriminant, $\Delta \delta \equiv \delta - \delta_0$, as a function of the temperature difference ΔT, calculated from Eq. (6) near the critical capillary radius: solid curve, water, $R = 0.47$ mm; dashed curve, ether, $R = 0.24$ mm. The domains of positive and negative δ correspond to the monotonic and oscillatory behaviors of the interface, respectively. (b) The contribution of phase change to the discriminant, $\Delta \delta \equiv \delta - \delta_0$, as a function of the temperature difference ΔT, calculated for water: dot-dashed curve, $R = 0.5$ mm; dashed curve, $R = 0.4$ mm; solid curve, $R = 0.3$ mm. (c) Same as (b) for diethyl ether: dot-dashed curve, $R = 0.4$ mm; dashed curve, $R = 0.3$ mm; solid curve, $R = 0.2$ mm. (d) Time evolution of the interfacial elevation h in transition from the monotonic dynamics to oscillations under the influence of evaporation, calculated for water with $R = 0.5$ mm. The curves are presented in terms of the normalized height h/H, calculated for each case separately: solid curve, $\Delta T = 0$ (no phase change); dashed curve, $\Delta T = 2$ K.
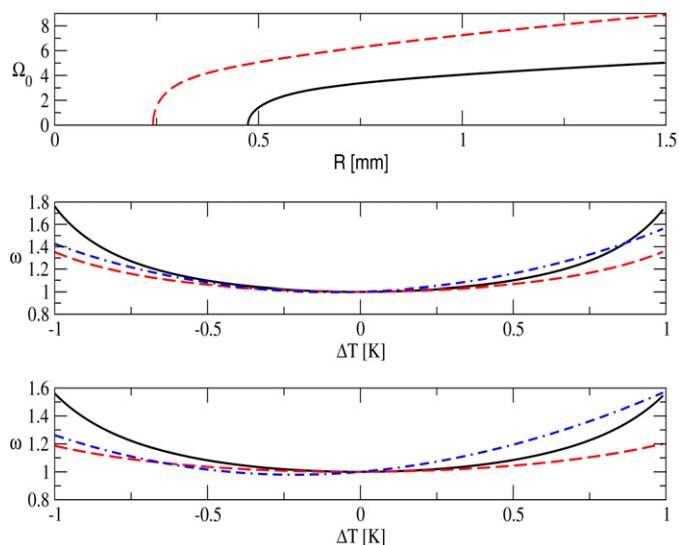
Fig. 9. (Upper panel) Oscillation frequency Ω_0 [Hz] as a function of the capillary radius R [mm] in the case of no interfacial mass transfer: solid curve, water; dashed curve, ether. (Middle panel) Variation of the normalized oscillation frequency $\omega = \Omega/\Omega_0$ with the temperature difference ΔT for water: solid curve, $R = 1.5$ mm; dashed curve, $R = 1$ mm; dot-dashed curve, $R = 0.5$ mm. (Lower panel) Variation of the normalized oscillation frequency $\omega = \Omega/\Omega_0$ with the temperature difference ΔT for ether: solid curve, $R = 1$ mm; dashed curve, $R = 0.5$ mm; dot-dashed curve, $R = 0.25$ mm.

The oscillation frequency, when oscillations are present, in the case of no phase change, Ω_0 is depicted as a function of the capillary radius R in Fig. 9 (upper panel). The effect of phase change on the normalized frequency $\omega = \Omega/\Omega_0$ of the oscillations, for water and ether in capillaries of various radii R, is shown in Fig. 9 (middle and lower panels). In general, both evaporation and condensation increase the frequency of the oscillation, since the amplitude of the oscillations is suppressed by vapor recoil, resulting in a shorter oscillation period. However, for capillaries with a radius near the critical radius corresponding to transition threshold from oscillatory to monotonic dynamics of the interface, the effect of mass gained by condensation causes a slight shift of the frequency curve. This is best seen for ether where under a relatively low rate of condensation, the frequency is first reduced before rising under the influence of recoil. This is yet another manifestation of the diverse stabilizing/destabilizing effect that condensation exerts on the flow. For a reasonable temperature jump of $\Delta T < 1$ K, this effect is only observable for capillaries of a small radius. Presumably, this is due to the fact that with an increase of the capillary radius, both the capillary driving force and the viscous drag decrease and the flow becomes more sensitive to recoil. In contrast, with high viscous damping and a dominant capillary force, the influence of mass loss becomes more apparent.

5. Summary

The Lucas–Washburn equation, describing the motion of a liquid body in a capillary tube is extended so as to account for the effect of interfacial mass transfer due to phase change—either evaporation or condensation. The system is shown to always possess a stable equilibrium state when the temperature jump is confined to a certain range. The augmented equation contains contributions related to mass loss/gain, which enter the viscous and inertial terms, as well as an additional effective force, vapor recoil which occurs as a result of the velocity jump at the interface. Our results show that several properties of the system behavior are altered due to the phase change: the equilibrium height, the transition from monotonic to oscillatory dynamics and the frequency of the oscillations, when present. Two mechanisms associated with phase change are mass loss/gain and vapor recoil. Mass gain/loss affect both the viscous and inertia terms. Vapor recoil results in a force directed into the liquid body for both evaporation and condensation. The diverse effect imparted by evaporation and condensation manifests through the mass loss/gain mechanism, in particular at lower rates of phase change. Evaporation is shown to always lower the equilibrium height, raise the oscillation frequency and lower the threshold for the onset of the oscillatory dynamics as the temperature difference across the interface increases. On the other hand, condensation is shown to have the opposite effect for low mass transfer rates, where the gained mass leads to a higher equilibrium height, lower frequency and a shift toward monotonic dynamics with an increase of the temperature jump across the interface. However, as the rate of condensation increases, this effect is overwhelmed by the vapor recoil force. At higher mass transfer rates and/or large capillary radii, vapor recoil is the dominant mechanism influencing the interface dynamics leading to a lower equilibrium height and higher oscillation frequencies for both evaporating and condensing menisci as compared to the isothermal case. The range of applicability of the present theory is limited in terms of the rate of phase change, as at its sufficiently high rates vapor recoil exceeds the capillary driving force and pushes the interface below zero level. The emergence of a negative or zero value for the meniscus height invalidates the Lucas–Washburn model framework.

Acknowledgments

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